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TECHNICAL MEMORANDUM

USE OF MAGNETIC TORGUE FOR POINTING CONTROL OF A SPINNING SKYLAB

Bellcomm

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TITLE- Use of Magnetic Torque for Pointing Control of a Spinning Skylab

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ABSTRACT

An artificial gravity experiment on Skylab B would require that the spin axis point to the sun. This memorandum concerns the use of magnetic torque for holding the spin axis aligned with the sun in the presence of gravity gradient bias torque and motion of the earth about the sun.

A pointing control law is formulated which uses sun sensor outputs to determine the magnetic moment to be developed by a main coil whose magnetic axis is coincident with the spin axis. The magnetic moment profile over the orbit approximates the profile requiring minimum electric energy from the power source.

Small vernier coils along the other two vehicle axes can be used to correct for misalignment between the main coil and the spin axis and to provide for bias momentum dumping when flying in the solar inertial attitude.

The magnetic control system requires the addition of amplifiers to control the current to the main and vernier coils, a magnetometer to measure the earth's magnetic field, and some computer capability. With an estimated 1700 watts available during the most demanding orbit of a selected 30 day experiment period, the magnetic system can be designed to weigh about 1450 lb.

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Introduction

It has been proposed that the Skylab B include an artificial gravity experiment implemented by spinning the spacecraft about its Z axis.* This axis should be held aligned to the earth-sun line in order to energize the solar cells during orbital day.

Without suitable control, both the gravity gradient bias torque and the apparent motion of the sun (solar precession) combine to cause an error in the spin axis-sun alignment. This memorandum develops a pointing attitude control system using magnetic torque and presents the magnetic system weight and power requirements for a spinning Skylab B.

Momentum Change Requirement

The magnetic control required to prevent a pointing error over one orbit can be expressed in terms of the momentum change that must be imparted to the vehicle to counteract the effect of both the gravity gradient torque and solar precession.

To cancel the effect of gravity gradient torque, the magnetic torque must provide, over one orbit, a momentum change of $\Delta \underline{H}_{gg}$ as shown in Figure 1. $\Delta \underline{H}_{gg}$ is the negative of the bias angular momentum that would be imparted to the vehicle in one orbit by gravity gradient torque. It lies in the orbital plane along a line perpendicular to the spin axis. Its magnitude and direction depend upon the magnitude and sign, respectively, of β, the angle from the orbital plane to the earth-sun line.

To compensate for solar precession, the spin axis must be shifted a small angle each orbit. The corresponding change in vehicle momentum $\Delta \underline{H}_{sp}$ is proportional to this angle and to the vehicle spin angular momentum. $\Delta \underline{H}_{sp}$ lies in the ecliptic plane and is perpendicular to the earth-sun line. It is in the direction of the apparent motion of the sun along the ecliptic and is practically constant in magnitude from orbit to orbit.

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*More precisely, the Z principal axis.

The momentum per orbit $\underline{H}_r$ which the magnetic control system must impart to the vehicle is then given by

$$\underline{H}_r = \Delta\underline{H}_{gg} + \Delta\underline{H}_{sp}$$

Its direction is perpendicular to the spin axis and its magnitude, for a given β, depends upon the angle ϕ between $\Delta\underline{H}_{gg}$ and $\Delta\underline{H}_{sp}$. The minimum value of ϕ for each β can be calculated, yielding a curve, Fig. 2*, representing the upper bound of $|\underline{H}_r|$ for Skylab B spinning at 6 rpm in a 235 nm, 50° inclination orbit.

Since β varies slowly with time during the mission, a 30 day period was selected such that the range of β encountered minimizes the required $|\underline{H}_r|$. For the period selected, β varies from 18° to -34°, requiring a magnetic control system which can develop 9200 ft-lb-sec per orbit at these extremes. For orbits with intermediate values of β, $|\underline{H}_r|$ is smaller.

Optimal Magnetic Control Law

A magnetic moment $\underline{M}$ established along the vehicle spin axis will react with the earth's magnetic field $\underline{B}$ to develop a torque $\underline{T}$ given by

$$\underline{T} = \underline{M} \times \underline{B} \qquad (1)$$

The torque will lie in a plane perpendicular to the spin axis, but its direction within the plane will vary during the orbital period due to the variation of $\underline{B}$ with orbital position. This suggests that with the proper control of the time variation of $|\underline{M}|$, the magnetic torque developed over the orbit can produce the required momentum change $\underline{H}_r$, which is also perpendicular to the spin axis. That is, it is desired to find $M(=|\underline{M}|)$ such that over the orbit

$$\int_0^T M\underline{m} \times \underline{B}\, dt = \underline{H}_r \qquad (2)$$

*A modification of Fig. 3, Ref. 1.

where M is the magnitude of the magnetic moment,

$\underline{m}$ is a unit vector in the direction of the spin axis,

$\underline{B}$ is the earth's magnetic field strength,

$\underline{H}_r$ is the required momentum change to be developed by the magnetic torque. It lies in the plane perpendicular to $\underline{m}$,

T = orbital period.

For a given magnet coil resistance, the electric power required to energize the magnet is proportional to M^2. A reasonable objective which yields a unique solution for M is one which minimizes $\int_0^T M^2 dt$, resulting in minimum electric energy per orbit.

The solution (See Appendix) is

$$M = -(\underline{B} \times \underline{m})' \left[\left(\int_0^T (\underline{B} \times \underline{m})(\underline{B} \times \underline{m})' dt \right)^{-1} \underline{H}_r \right] \tag{3}$$

The vector within the brackets is a constant for the orbit; its evaluation requires knowledge of $\underline{B}$ and $\underline{H}_r$ for each orbit before it is flown. M varies over the orbit due to variations in the factor $(\underline{B} \times \underline{m})'$. Because $\underline{B}$ and $\underline{H}_r$ are not precisely known in advance, this optimal control law is difficult to implement. However, it does suggest the more practical control law given below.

<u>Magnetic Moment Pointing Control Law</u>

Let M be defined over the n^{th} orbit by

$$M_n = -(\underline{B}\times\underline{m})' K\underline{H}_n \quad ; \quad K = \text{scalar factor} \tag{4}$$

Let $K\underline{H}_n$ be set for the first orbit equal to the vector determined by evaluating, for that orbit, the bracketed expression in Eq. (3). Then exactly $\underline{H}_r$ is developed during the first orbit. Let $\underline{H}_n$ be incremented at the start of each succeeding orbit by

$$\Delta\underline{H}_{n-1} = \left[\underline{H}_r - \int_0^T \underline{M} \times \underline{B}\, dt\right]_{n-1} ,$$

that is, the difference between the required and developed momentum of the previous orbit. Then,

$$\underline{H}_n = \underline{H}_{n-1} + \Delta\underline{H}_{n-1}$$

A computer simulation demonstrates that if this process is continued for several orbits, then on the average

$$\Delta\underline{H}_n = \underline{0}$$

For some orbits the developed momentum $\int_0^T \underline{M} \times \underline{B}\, dt$ is more than the required momentum $\underline{H}_r$, and other orbits less, but on the average they equalize. Orbit by orbit, $\underline{H}_n$ adjusts itself to provide the proper $\underline{M}$ to keep $\underline{H}_n$ within a small bounded region.

It was found that a wide range of scale factor K is tolerable provided that $K\underline{H}_n$ is properly initialized for the first orbit. However, the computer simulation demonstrated that even if $\underline{H}_n$ were initialized to the null vector, a value of K ranging from 1.0×10^4 to 1.5×10^4 (with $\underline{H}_n$ given in ft-lb-sec, $\underline{B}$ in lines/in.^{2}, and $\underline{M}$ in amp-turn-in.^{2}) produced after a few orbits, the same $\underline{M}$ as when $\underline{H}_n$ had been properly initialized.

Observe that with $\underline{H}_0 = \underline{0}$,

$$\underline{H}_n = \sum_{i=1}^{n-1} \underline{H}_{r,i} - \int_0^{(n-1)T} \underline{M} \times \underline{B}\, dt \qquad (5)$$

That is, $\underline{H}_n$ is the difference, from initiation of the control law to the beginning of the n^{th} orbit, between the sum of the required momentum and the developed momentum. It is, in effect, the error in momentum, $\underline{H}_{error}$, applied to the vehicle, a fact that will now be used for determining $\underline{H}_n$ without the need of evaluating Eq. (5).

Fig. 3 shows the vehicle axes and the spin angular momentum vector $\underline{H}_s$ coincident with the spin axis. If the spin axis initially points to the sun and a measurement at the start of the n^{th} orbit shows pointing angle errors of ψ and θ about the Y and X axes, then for small error angles

$$\underline{H}_{error} = \underline{H}_n = H_s \begin{pmatrix} \psi \\ \theta \\ 0 \end{pmatrix}_n \qquad (6)$$

where H_s is the magnitude of the spin angular momentum $\underline{H}_s$. Substituting (6) into (4) yields the magnetic moment pointing control law

$$M_n = -(\underline{B} \times \underline{m})' K H_s \begin{pmatrix} \psi \\ \theta \\ 0 \end{pmatrix}_n \qquad (7)$$

Application of the above control law results in nearly constant errors in ψ and θ from orbit to orbit. These errors can be decreased by redefining $\underline{H}_n$ as

$$\underline{H}_n = \underline{H}_{n-1} + K_1 H_s \begin{pmatrix} \psi \\ \theta \\ 0 \end{pmatrix}_n , \quad K_1 < 1.0 \qquad (8)$$

With this modification, the presence of an angle error serves to alter $\underline{H}_n$ and M on the following orbit, thus tending to diminish the error.

Control Law Simulation

To obtain an estimate of the magnetic moments required by the pointing control law, Eq. (4) was simulated by incrementing $\underline{H}_n$ by the difference between the required and developed momentum of the previous orbit. That is,

$$\underline{H}_n = \underline{H}_{n-1} + \left[\underline{H}_r - \int_0^T \underline{M} \times \underline{B} \, dt \right]_{n-1}$$

With $\underline{H}_r$ = (9200, 0, 0) ft-lb-sec, obtained from Fig. 3, the largest rms value of magnetic moment required over one orbit during one earth day was 2.82×10^8 amp-turn-in^2. This magnetic moment is to be developed by a coil (called the main coil) wrapped around the perimeter of the Workshop with its magnetic axis along the spin axis.

If magnetic torque is used for pointing control of the spinning Skylab, then it would be prudent to also use magnetic torque for dumping bias momentum when the Skylab is flying solar inertial.

Simulation of the $\underline{M} = K(\underline{H}_s \times \underline{B})$* control law for this mode yields a rms magnetic moment requirement of 8.2×10^6 amp-turn-in.^{2}, but not necessarily along the spin axis. The components along the other two axes can be provided by vernier coils whose magnetic moments are directed along those axes.

These vernier coils can also be used during the spinning mode to correct misalignment between the main coil magnetic axis and the spin axis. This correction is desirable because any misalignment produces a component of magnetic torque which alters the spin angular velocity. With the magnetic moment values given above, the vernier coils can correct up to 1-1/2° of main coil misalignment.

Magnetic Design

The magnetic control system to execute the magnetic moment pointing control law requires a main coil, two vernier coils, a control amplifier to control the currents to the coils,

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*Magnetic control law for dumping bias momentum, Ref. 2.

a magnetometer to measure the earth's magnetic field, the sun sensor output to determine pointing errors and some computer capability.

The coil design offers a trade-off between coil weight and coil power, and depends upon the choice of conductor cross sectional area. The following relationships apply.

Coil conductor weight = $\sigma \ell N a$ lbs

$$\text{Coil average power} = \frac{M^2 \ell \rho}{A^2 N a} \text{ watts}$$

where

σ = conductor specific weight, lbs/in.^{3}

ℓ = length per coil turn, in.

N = number of turns

a = conductor cross sectional area, in.^{2}

M = rms magnetic moment, amp-turn-in.^{2}

A = area enclosed by coil, in.^{2}

ρ = conductor resistivity, ohms/in.^{2}

The coil power is the average power over the orbit for which M is determined.

The main coil is wrapped around the perimeter of the Workshop and has a rectangular shape with sides 700 in. and 240 in. in length. The vernier coils are 20 ft diameter coils and can be located in the aft end of the Workshop. All coils are made of aluminum conductor.

To minimize the coil weights, they are designed here to utilize all the available power. At β = 18°, 1700 watts are available* for magnetic control, some of which is diverted from experiments not conducted during the spinning mode. The resulting weights and average power during the worst orbit are shown in Table 1. The coil weights include the coil insulation.

*Interpolated from data supplied by B. W. Moss.

Table 1

Main Coil	Weight (lb)	Av. Power (watts)
Coil	1185	1525
Control Amplifier	183	80
Vernier Coils		
Both Coils	65	90
Control Amplifier	21	5
	1454	1700

If RCS fuel had been used instead of magnetic torque to provide pointing control during the same 30 day period, 364 pounds of fuel would have been consumed. Hence the additional weight of the magnetic control system is in effect 1090 pounds. However, the magnetic control system can also provide for bias momentum dumping during the solar inertial mode with a maximum average power over an orbit of 95 watts.

Summary

Solar pointing control of a spinning Skylab B can be accomplished by a magnetic torquing coil whose magnetic axis is coincident with the vehicle spin axis.

A control law is suggested for determining the required magnitude of the magnetic moment. It is based on measurements, once per orbit, of solar pointing errors, and yields the magnetic moment profile requiring near minimum electric energy from the power source.

A 30 day experiment period has been selected such that the range of β encountered minimizes the control requirements. A computer simulation, using the known momentum change per orbit to be imparted to the spin axis, yields the magnetic moment which the coil must be prepared to supply.

In addition to the main coil for pointing control during the spinning mode, small vernier coils can be provided to correct misalignment between the main coil and the spin axis, and also to provide for bias momentum dumping during the solar inertial mode.

With an estimated 1700 watts available for magnetic control, the magnetic system consisting of main and vernier coils and their associated control amplifiers is estimated to weigh 1454 lb.

Acknowledgement

Jack Kranton suggested the use of a single coil for magnetic control of the spinning Skylab.

W. Levidow

1022-WL-cf W. Levidow

Attachments
Appendix
Figures 1-3

REFERENCES

1. Ravera, R. J. and Hough, W. W., "Estimate of Daily Fuel Required to Control Gravity Gradient and Solar Precession during an Artificial G Experiment," Bellcomm Memorandum for File, June 15, 1970.

2. Levidow, W., "Use of Magnetic Torque for CMG Momentum Management," Bellcomm Technical Memorandum, TM-69-1022-8, December 2, 1969.

APPENDIX

Minimum Energy Magnetic Moment

Problem: Minimize $1/2 \int_0^t M^2 \, dt$

subject to the constraint

$$\int_0^t M\underline{m} \times \underline{B} \, dt = \underline{H} \qquad \text{(A-1)}$$

where $\underline{m} \cdot \underline{H} = \underline{0}$

Solution:

Let $G = \int_0^t \left[1/2 \, M^2 + M(\tilde{\underline{B}} \, \underline{m})' \underline{\lambda} \right] dt$

where $\tilde{\underline{B}}$ is the matrix equivalent to the vector cross product operation $\underline{B}\times$.

$\underline{\lambda}$ is a vector of Lagrange multipliers.

Differentiating the integrand in G with respect to M and equating the resulting expression to zero yields

$$M = -(\tilde{\underline{B}} \, \underline{m})' \underline{\lambda} \qquad \text{(A-2)}$$

Substituting into (A-1)

$$\underline{\lambda} = \left[\int_0^t \tilde{\underline{B}} \, \underline{m} \, (\tilde{\underline{B}} \, \underline{m})' \, dt \right]^{-1} \underline{H}$$

Substituting into (A-2)

$$M = -(\tilde{\underline{B}} \, \underline{m})' \left[\int_0^t \tilde{\underline{B}} \, \underline{m} \, (\tilde{\underline{B}} \, \underline{m})' \, dt \right]^{-1} \underline{H} \qquad \text{(A-3)}$$

The solution (A-3) requires that the matrix within the brackets be non-singular. Let $\tilde{\underline{B}}\ \underline{m} = \underline{S}$, a vector in the two-space defined by the plane perpendicular to $\underline{m}$.

The matrix

$$\int_0^t \tilde{\underline{B}}\ \underline{m}\ (\tilde{\underline{B}}\ \underline{m})'dt = \int_0^t \underline{S}\ \underline{S}'\ dt$$

is positive if

$$Q = \underline{U}' \int_0^t \underline{S}\ \underline{S}'\ dt\ \underline{U}$$

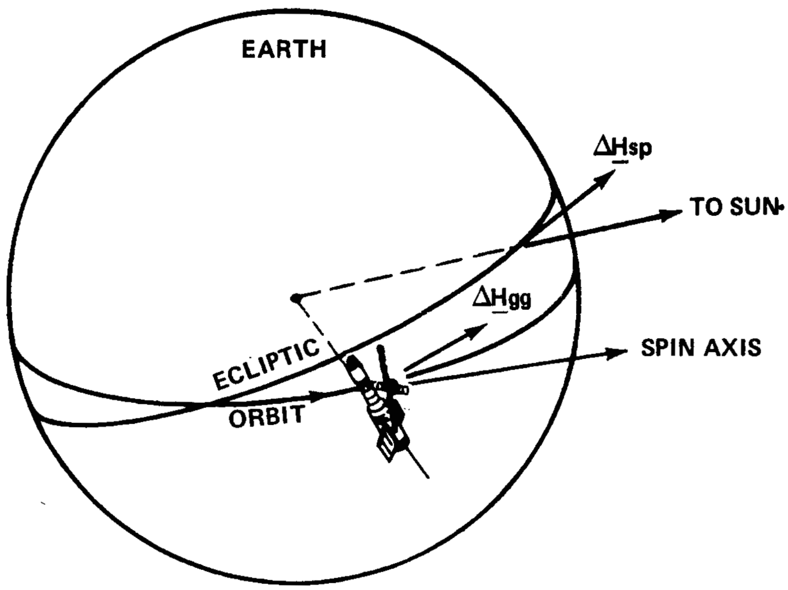
is positive definite, where $\underline{U}$ is an arbitrary time invariant vector in the same two-space as $\underline{S}$

$$Q = \int_0^t \underline{U}'\ \underline{S}\ \underline{S}'\ \underline{U}\ dt$$

$$\int_0^t |\underline{U}|^2\ |\underline{S}|^2\ \cos^2\theta\ dt$$

where θ is the angle between $\underline{U}$ and $\underline{S}$.

Since $\underline{U}$ is fixed but $\underline{S}$ varies in direction with time, $\cos\theta$ cannot be zero over the entire time interval [0, t]. Hence the matrix $\int_0^t \tilde{\underline{B}}\ \underline{m}\ (\tilde{\underline{B}}\ \underline{m})'dt$ is positive definite and therefore non-singular.



ΔHgg = GRAVITY GRADIENT TORQUE MOMENTUM CORRECTION

ΔHsp = SOLAR PRECESSION MOMENTUM CORRECTION

FIGURE 1 - PER ORBIT MOMENTUM CHANGE COMPONENTS

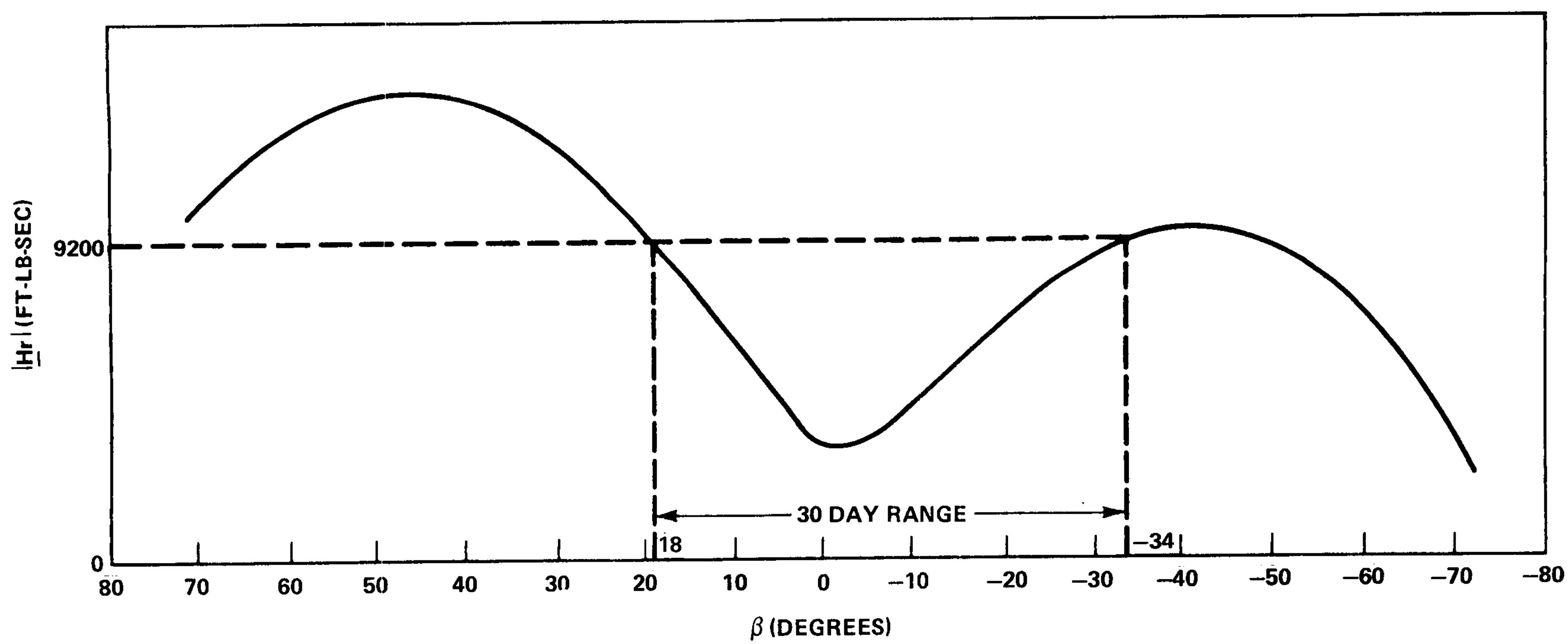


FIGURE 2 - MOMENTUM CHANGE REQUIRED PER ORBIT FOR SOLAR POINTING CONTROL
6 RPM, 235 NM, 50° ORBIT

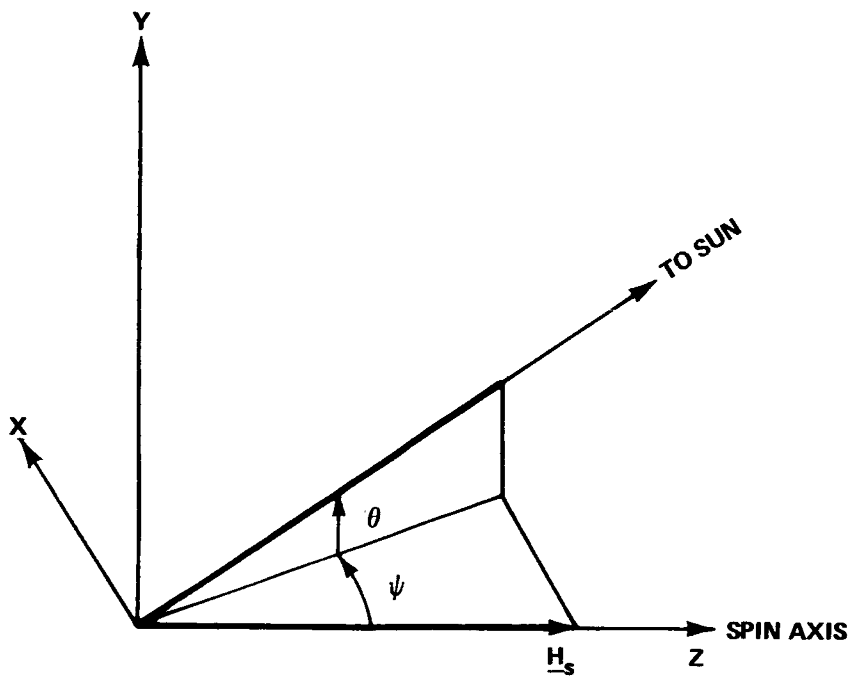


FIGURE 3 - SOLAR POINTING ANGLE ERRORS